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False Vacuum Decay with Gravity in a Critical Case

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The vacuum decay in a de Sitter universe is studied for the class of effective inflaton potentials that curvature at the top is less than as well as greater than a critical value determined previously. By comparing the actions of the Hawking - Moss instanton and the Coleman - de Luccia instanton(s) the mode of vacuum decay is determined in this critical situation.

KEY WORDS: vacuum decay; instanton.

1. INTRODUCTION

The idea of vacuum decay in a de Sitter universe was developed by Coleman and de Luccia in Coleman and de Luccia (1980) and plays an important role in the cosmological inflationary scenario. It is considered as a mechanism of transition to a Friedman universe in old inflation (Guth, 1981) and emerges also in the scenario of open inflation (Bucher *et al.*, 1995; Linde and Mezhlumian, 1995; Kofman *et al.*, 1997, 1994).

We consider single scalar field Φ with self-interaction given by the nonnegative function $V(\Phi)$ - effective potential - that has two nondegenerate minima, one of them strictly positive (false vacuum) and second one equal to zero (true vacuum). These vacua are supposed to be separated by a finite potential barrier. Let V reach its local maximum V_M at Φ_M. Furthermore, let us denote by $H(\Phi)$ the Hubble parameter corresponding to the energy-density $V(\Phi)$: $H(\Phi) = \sqrt{8\pi V(\Phi)/3}$, especially $H_M = \sqrt{8\pi V_M/3}$. In order to study quantum transition of the inflaton, in fact, one does not need to have the potential described above, namely the potential may have no local minima (vacua), since the existence of potential barrier is sufficient for this purpose. Supposing $O(4)$ symmetry supplemented by the regularity of the solution we get the following (Euclidean) equations of motion and boundary conditions for Euclidean version of the scale parameter a and the

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inflaton Φ

$$a'' = -C\left(\Phi'^2 + V\right)a, \quad \Phi'' + 3\frac{a'}{a}\Phi' = V'_\Phi \tag{1}$$

$$a(0) = 0, \quad a'(0) = 1, \quad \Phi'(0) = \Phi'\left(\tau_f\right) = 0, \tag{2}$$

where the constant C equals $8\pi/3$ and $\tau_f > 0$ is defined by the equation $a(\tau_f) = 0$. For any suitable potential there exists a trivial solution of the above problem that reads

$$\Phi_{HM} = \Phi_M, \quad a_{HM} = H_M^{-1}\sin\left(H_M\tau\right), \quad \text{with} \quad \tau_f = \frac{\pi}{H_M} \tag{3}$$

and is called the Hawking-Moss instanton (Hawking and Moss, 1982). This instanton mediates the vacuum decay in such a way that the inflaton jumps up to the top of the barrier in the horizon-size domain and afterwards the inflaton leaves (by quantum or thermal fluctuations) the unstable equilibrium and evolves classically to the true vacuum. However, under some additional conditions, the problem (1) and (2) has also nontrivial solutions (with variable Φ) called Coleman-de Luccia (CdL) instantons (or bounces) (Coleman and de Luccia, 1980). Following the ideas of paper (Balek and Demetrian, 2005) (see also Tanaka, 1999 and recently Hackworth and Weinberg, 2005) and Weinberg (2005) CdL instantons can be characterized by how many times the inflaton crosses the top of the barrier. We talk about the CdL instanton of the lth order if the inflaton crosses the top l-times.

The boundary conditions (2) provide the action of a CdL instanton (that follows from the general Einstein-Hilbert action) to be finite. The action is a very important quantity for an instanton since it determines the probability of the vacuum decay per unit space-time volume in the form $\exp(-S)$. This quantity can be transformed, according to (Balek and Demetrian, 2005), into the following simple form

$$S = 2\pi^2\int_0^{\tau_f}\left[\left(\frac{1}{2}\Phi'^2 + V\right)a^2 - \frac{1}{C}\left(aa'^2 + 1\right)\right]a\,d\tau = -\frac{4\pi^2}{3C}\int_0^{\tau_f} a\,d\tau. \tag{4}$$

It is easy to find that the action of the Hawking - Moss instanton is given by

$$S_{HM} = -\frac{\pi}{H_M^2}. \tag{5}$$

2. NEAR-TO-LIMIT CdL INSTANTON OF THE FIRST ORDER AND ITS ACTION

As it was sketched in Jensen and Steinhardt (1984) and finally proved in Balek and Demetrian (2004) the CdL instanton necessarily exists for potentials with $V''_M/H_M^2 < -4$ and may exist if $V''/H^2 < -4$ for some value of Φ in the potential barrier. If the fraction V''_M/H^2_M approaches one of the values $-l(l+3)$, $l = 1, 2, 3\ldots$, the near-to-limit CdL instanton (that approaches necessarily existing Hawking-Moss instanton) may exist. If $V''_M/H^2_M < -l(l+3)$ then the instanton of lth order necessarily exists. Our task is to compute the difference between the action of the near-to-limit CdL instanton of the first order and the action of the related Hawking-Moss instanton. This task has been considered in Balek and Demetrian (2005), but our treatment will be different. By making use of the re-scaled Euclidean time $x = H_M\tau$ and the shifted inflaton field $y = y(x) = \Phi(\tau(x)) - \Phi_M$ we rewrite equations (1) in to the form

$$a'' = -C\left(y'^2 + \frac{V}{H^2_M}\right)a, \quad y'' + 3\frac{a'}{a}y' = \frac{V'_y}{H^2_M}. \tag{6}$$

Since now the prime denotes differentiation with respect to x. Introducing the expansion of the relevant quantities, including the dimensionless Euclidean action

$$\sigma = -\frac{3CH^2_M}{4\pi^2}S, \qquad (\sigma_{HM} = 2)$$

into the series in the inflaton amplitude k

$$y(x) = \sum k^n u_n(x), -\frac{V''_M}{H^2_M} = 4 + \sum k^n \Delta_n,$$
$$a(x) = CH^{-1}_M \sum k^n v_n(x), \quad \sigma = 2 + \sum k^n w_n \tag{7}$$

and expanding the potential into the powers of y we replace the nonlinear equations (6) by the infinite system of linear equations

$$u''_n(x) + 3\frac{\cos(x)}{\sin(x)}u'_n(x) + 4u_n(x) = \mathcal{U}_n(x),$$
$$v''_n(x) + v_n(x) = \mathcal{V}_n(x)\sin(x) \tag{8}$$

in which the functions $\mathcal{U}_n$ and $\mathcal{V}_n$ can be computed order by order if we know the functions $u_{n-1}, u_{n-2}, \ldots, u_0$. Functions u_n and v_n are defined on the interval $[0, x^{(n)}_f]$, where $x^{(n)}_f$ is defined as the point in which the scale factor a computed up to the nth order in k vanishes. The value of the functions v_n and v'_n must vanish at $x = 0$ (this follows from (2)) and u_n must be regular. We know that $u_0 = 0$, $v_0 = \sin(x)$ and $u_1 = \cos(x)$. Furthermore, $V'_M = 0$ implies that v_1 vanishes. The next nonzero term in the k-expansion of a is given by v_2 that must obey the following equation:

$$v''_2 + v_2 = -\frac{1}{4}\left[\sin(x) - 3\sin(3x)\right],$$

which solution reads

$$v_2(x) = \frac{1}{4}\left[\frac{5}{8}\sin(x) + \frac{1}{2}x\cos(x) - \frac{3}{8}\sin(3x)\right]. \tag{9}$$

Solving equation $v_0(x) + k^2 v_2(x) = 0$ with the accuracy up to the order k^2 and supposing the solution is close $x = \pi$, we find that the shifted right end-point is given by

$$x_f^{(2)} = \pi - \frac{1}{8}C\pi k^2 \equiv \pi + \delta^{(2)}.$$

Knowing v_2 we can compute the contribution of the order k^2 to the difference between the actions of CdL and Hawking - Moss instanton that is defined by Eqs. (4) and (7). The result is: $w_2 = \int_0^\pi v_2(x)\,dx = 0$. This means that we cannot distinguish between the action of a near-to-limit CdL instanton and the related Hawking - Moss instanton in the second order of inflaton amplitude and we must continue our computations. Equation for u_2 reads

$$u_2'' + 3\frac{\cos(x)}{\sin(x)}u_2' + 4u_2 = \frac{1}{2}\frac{V_M'''}{H_M^2}\cos^2(x)$$

and its regular solution is

$$u_2(x) = \frac{1}{24}\frac{V_M'''}{H_M^2}[1 - 2\cos^2(x)]. \tag{10}$$

Now, we can derive equation for v_3 and its solution

$$\begin{aligned} v_3'' + v_3 &= \frac{V_M'''}{48H_M^2}\,[2\sin(2x) - 5\sin(4x)] \;\Rightarrow v_3 \\ &= -\frac{V_M'''}{72H_M^2}\left[\sin(2x) - \frac{1}{2}\sin(4x)\right]. \end{aligned} \tag{11}$$

There is no shift of the right end point x_f since $v_3(\pi) = 0$, and we easily find that the k^3-contribution to the action (w_3) equals zero. This result forces us to continue up to the fourth order in k. The shift of the right end point x_f with respect to π (which is actually of the order k^2) must be taken under consideration in the equation for u_3. Introducing new a independent variable

$$X = \frac{\pi x}{\pi + \delta^{(2)}} = x\left(1 + \frac{1}{8}Ck^2 + o(k^2)\right) \equiv Kx$$

we derive the equation for u_3 of the form

$$\frac{d^2u_3(X)}{dX^2} + 3\frac{\cos(X)}{\sin(X)}\frac{du_3(X)}{dX} - \frac{V_M''}{H_M^2}u_3(X)$$

$$
\begin{aligned}
&= \left[\frac{1}{K^2}\left(4+\frac{V''_M}{H_M^2}\right)+\frac{13}{4}C+\frac{1}{24}\left(\frac{V'''_M}{H_M^2}\right)^2\right]\cos(X) \\
&\quad + \left[\frac{1}{6}\frac{V''''_M}{H_M^2}-\frac{9}{4}C-\frac{1}{12}\left(\frac{V'''_M}{H_M^2}\right)^2\right]\cos^3(X) \\
&\equiv A\cos(X)+B\cos^3(X).
\end{aligned}
$$

The only regular $\left(\frac{du_3(0)}{dX}=\frac{du_3(\pi)}{dX}=0\right)$ solution to this equation is given by: $u_3(X)=\beta\cos^3(X)$, with the constant β to be determined from the system of linear equation

$$
6\beta = A,\ -14\beta = B \ \Rightarrow\ \beta = -\frac{1}{14}\left\{\frac{1}{6}\frac{V''''_M}{H_M^2}-\frac{9}{4}C-\frac{1}{12}\left(\frac{V'''_M}{H_M^2}\right)^2\right\}.
$$

However, the fixation of β is only a supplementary consequence of previous system of linear equations, since their main purpose is to determine the value of k^2 as a function of A and B. Namely, we obtain from them the following "quantization rule" for k^2 as a function of $4+V''_M/H_M^2$

$$
k^2 = -\frac{4+\frac{V''_M}{H_M^2}}{\frac{2}{7}\left[8C+\frac{1}{48}\left(\frac{V'''_M}{H_M^2}\right)^2+\frac{1}{4}\frac{V''''_M}{H_M^2}\right]}. \tag{12}
$$

If the denominator of the fraction on the right hand side is positive then we have, for small negative numerator a near-to-limit CdL instanton of the first order whose inflaton amplitude is given by (12). We will return to the case when the denominator is negative later. Now, let us concentrate on computation of the action of this near-to-limit CdL instanton. Performing some tedious algebra one derives equation for v_4 of the form

$$
v''_4 + v_4 = \left[\aleph_0+\aleph_2\cos^2(x)+\aleph_4\cos^4(x)\right]\sin(x),
$$

where we have introduced the parameters

$$
\begin{aligned}
\aleph_0 &= -\frac{15}{16}C+\frac{1}{288}\left(\frac{V'''_M}{H_M^2}\right)^2 \\
\aleph_2 &= \frac{159}{224}C-\frac{2}{21}\left(\frac{V'''_M}{H_M^2}\right)^2+\frac{3}{28}\frac{V''''_M}{H_M^2} \\
\aleph_4 &= \frac{27}{56}C+\frac{1}{7}\left(\frac{V'''_M}{H_M^2}\right)^2-\frac{9}{56}\frac{V''''_M}{H_M^2}.
\end{aligned}
$$

And finally, the solution is

$$
\begin{aligned}
v_4 = \frac{1}{192}\{&-12(8\aleph_0 + 2\aleph_2 + \aleph_4)x\cos(x)\\
&+\sin(x)[96\aleph_0 + 36\aleph_2 + 23\aleph_4 - 2(6\aleph_2 + 5\aleph_4)\cos(2x)\\
&-2\aleph_4\cos(4x)]\}.
\end{aligned} \tag{13}
$$

We finish the computations with a nonzero contribution to the action of a surprisingly simple form

$$
\Delta S^{(4)} \equiv -\frac{4\pi^2 k^4 w_4}{3CH_M^2} = \frac{2\pi^2}{15}\frac{k^2}{H_M^2}\left(4 + \frac{V_M''}{H_M^2}\right). \tag{14}
$$

Formula (14) tells us that a near-to-limit CdL instanton of the first order has, in the case $V_M''/H_M^2 < -4$, less action than the related Hawking - Moss instanton and therefore, if no other instantons exist, it is the instanton governing the false vacuum decay.

Let us demonstrate the power of the formulas (12) and (14) on a concrete example. We will consider the often mentioned quartic potential

$$
V(\Phi) = \frac{1}{2}\Phi^2 - \frac{1}{3}\delta\Phi^3 + \frac{1}{4}\lambda\Phi^4, \tag{15}
$$

where δ and λ are supposed to be positive. The non-negativeness of the potential and the existence of the false vacuum (the true vacuum is located at $\Phi = 0$) require that the δ parameter belongs to the interval (δ_m, δ_M), where $\delta_m = 2\sqrt{\lambda}$, $\delta_M = 3\sqrt{\lambda/2} \approx 1.06\delta_m$. The positions of the false vacuum (Φ_{fv}) and of the top of the barrier are given by

$$
\Phi_{fv} = \frac{\delta}{2\lambda} + \sqrt{\frac{\delta^2}{4\lambda^2} - \frac{1}{\lambda}} = \frac{\sqrt{1-Z^2}}{(1-Z)\sqrt{\lambda}},
$$

$$
\Phi_M = \frac{\delta}{2\lambda} - \sqrt{\frac{\delta^2}{4\lambda^2} - \frac{1}{\lambda}} = \frac{\sqrt{1-Z^2}}{(1+Z)\sqrt{\lambda}},
$$

where

$$
Z = \sqrt{1 - \frac{4\lambda}{\delta^2}}, \quad Z \in \left[0, \frac{1}{3}\right].
$$

The potential (15) in the (Z, λ) parametrization has the form

$$
V = \frac{1}{2}\Phi^2 - \frac{2}{3}\frac{\sqrt{\lambda}}{\sqrt{1-Z^2}}\Phi^3 + \frac{1}{4}\lambda\Phi^4.
$$

We are interested in the quantities

$$H_M^2 \equiv \frac{8\pi}{3} V_M = \frac{2\pi}{9\lambda} \frac{(1-Z)(1+3Z)}{(1+Z)^2}, \quad V_M'' = -\frac{2Z}{1+Z}.$$

From these expressions it follows that the effective curvature of the potential at its top is given by

$$\frac{V_M''}{H_M^2} = -\frac{9\lambda}{\pi} \frac{Z(1+Z)}{(1-Z)(1+3Z)} \tag{16}$$

and is a monotonically increasing function of Z. If we denote by λ_S the value of λ at which (at given Z) $V_M''/H_M^2 = -4$, then

$$\lambda_S = \frac{4\pi}{9} \frac{(1-Z)(1+3Z)}{Z(1+Z)}.$$

It will be useful to express the fractions V_M'''/H_M^2 and V_M''''/H_M^2 entering the formulae (12) and (14) in terms of the parameters of the quartic potential. After some algebra one finds out that

$$\frac{V_M'''}{H_M^2} = -\frac{9\lambda^{3/2}}{2\pi} \frac{(1+Z)^2}{(1-Z)\sqrt{1-Z^2}}, \quad \text{and} \quad \frac{V_M''''}{H_M^2} = \frac{27\lambda^2}{\pi} \frac{(1+Z)^2}{(1-Z)(1+3Z)}$$

and by using the relation (16) one gets the dependence of the fractions in question on the effective curvature V_M''/H_M^2 of the potential at its top.

$$\begin{aligned} \frac{V_M'''}{H_M^2} &= \frac{\sqrt{\pi}}{6} \left(\frac{1+Z}{1-Z}\right)^{1/2} \left(\frac{1}{Z}+3\right)^{3/2} \left(-\frac{V_M''}{H_M^2}\right)^{3/2}, \\ \frac{V_M''''}{H_M^2} &= \frac{\pi}{3} \frac{(1-Z)(1+3Z)}{Z^2} \left(\frac{V_M''}{H_M^2}\right)^2. \end{aligned} \tag{17}$$

Now, we are ready to compare the predictions of the formulas (12) and (14) with the numerical solutions of exact instanton Equation (6). In order to perform the numerical analysis of the instanton equations we will fix the parameter Z of the quartic potential to have the value corresponding to central point between the case when the false vacuum energy density V_{fv} is negligible in comparison to V_M (this situation corresponds to the thin-wall approximation considered in Coleman and de Luccia (1980) and is analyzed in part numerically in Samuel and Hiscock (1991)), and the case when the false vacuum disappears. Since the energy density in false vacuum is given by

$$V_{fv} = \frac{1}{12\lambda} \frac{(1+Z)(1-3Z)}{(1-Z)^2},$$

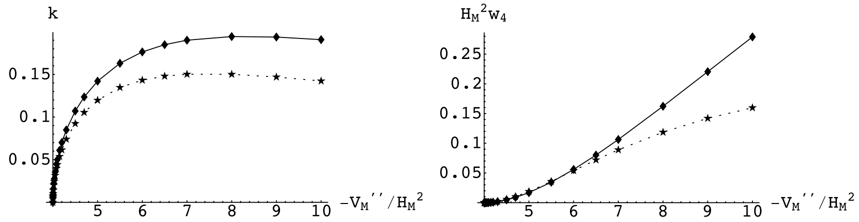

Fig. 1. The prediction of the analytical formula (12) for the instanton width in Φ direction (lower, doted, line) is compared with the numerical computations of this quantity in the left graph. The range of $-V''_M/H^2_M$ is taken (Linde and Mezhlumian, 1995; Demetrian, 2004); at the value 4 the CdL instanton of the first order appears and at the value approximately 10 the second order CdL instantons appear (Balek and Demetrian, 2004; Demetrian, 2004). Finally, the right graph shows the theoretical dependence of the first-order CdL instanton action according to (14) (lower, doted, line) together with numerically obtained values of this quantity.

the fraction V_{fv}/V_M, that depends on Z only, equals 1/2 if

$$\left(\frac{1+Z}{1-Z}\right)^3 \frac{1-3Z}{1+3Z} = \frac{1}{2} \quad \text{with} \quad Z \in [0, 1/3].$$

This equation determines Z as

$$Z \approx 0.278 \,. \tag{18}$$

We have solved numerically the exact instanton equations with this choice of the parameter Z and compared these results with the approximative formulae (12) and (14), see Fig. 1.

3. CdL INSTANTON(S) OF THE FIRST ORDER IN THE CASE WITH SUBCRITICAL VALUE OF THE FOURTH DERIVATIVE OF THE EFFECTIVE POTENTIAL AT ITS TOP

Let us consider an effective potential which has, for a suitable choice of parameters, such a shape that the denominator in the formula (12) is negative and at the same time it is possible to change continuously the sign of the nominator. If the sign of the term $-4 - V''_M/H^2_M$ is positive, then there must be at least one CdL instanton of the first order, as discussed previously. But what happens when we pass through zero to negative values of $-4 - V''_M/H^2_M$, keeping $[8C + \frac{1}{48}(\frac{V'''_M}{H^2_M})^2 + \frac{1}{4}\frac{V''''_M}{H^2_M}]$ a negative constant? The formula (12) ensures that we have the near-to-limit CdL instantons in the region with $-V_M/H^2_M$ (a little bit) less than 4. By the continuity argument, this set of instantons must be lined-up to the "overcritical" instantons existing for $-V''_M/H^2_M > 4$. In order to investigate the structure of the instanton solutions it is helpful to use the method of representation of a CdL instanton proposed by Tanaka in Tanaka (1999). Let us consider the two dimensional “phase” plane (Π, Φ), where Φ stands for some value (to be determined later) of the inflaton and Π stands for some value of the conjugated momentum $2\pi^2 a^3 \Phi'$. For a given V we can start the evolution, using to the Euclidean equations of motion (1), (2) for a and Φ, with any initial value Φ_i of the inflaton and we will come, in some finite Euclidean time $\bar{\tau}(\Phi_i)$, to the point at which a reaches its maximum. Let Φ_i^+ be an arbitrary value of Φ located to the right of Φ_M. Taking this Φ_i^+ as the initial value for the system (1), (2) we obtain the point $(\bar{\Pi}^+, \bar{\Phi}^+) \equiv (\Pi(\bar{\tau}), \Phi(\bar{\tau}))$, and varying Φ_i^+ we can draw the curve

$$\mathcal{C}^+ = \{(\bar{\Pi}^+, \bar{\Phi}^+),\ \Phi_i^+ > \Phi_M\}.$$

Analogically, varying the initial value of the inflaton Φ_i^- located to the left of Φ_M we construct the curve $\mathcal{C}^-$. Intersections of the curves $\mathcal{C}^+$ and $\mathcal{C}^-$ correspond to the CdL instantons. (The curve $\mathcal{C}^+$ does not intersect itself and the same holds for $\mathcal{C}^-$).

The existence of two CdL instantons of the first order for given $-V_M''/H_M^2$ opens the question which instanton governs the vacuum decay. Let us investigate a concrete realization of the kind of vacuum decay described above. In the appendix it is shown that for a class of generalizations of the quartic potential we cannot obtain negative value of the fourth derivative of the potential at Φ_M if we require that the potential contains both the false and true vacuum. Let us relax these requirements and consider the potential

$$V(\Phi) = \frac{3}{8\pi} - \frac{1}{2} g_2 \Phi^2 - \frac{1}{24} g_4 \Phi^4 \,, \tag{19}$$

where g_2, g_4 are positive constants. The top of the potential is at $\Phi_M = 0$ and

$$V_M'' = -g_2, \qquad H_M^2 = 1\,, \qquad -\frac{V_M''}{H_M^2} = g_2.$$

The choice

$$g_4 = 300$$

ensures that

$$8C + \frac{1}{48}\left(\frac{V_M'''}{H_M^2}\right)^2 + \frac{1}{4}\frac{V_M''''}{H_M^2} = 8C - \frac{1}{4} g_4$$

is negative. We have performed numerical analysis of the structure of CdL instanton solutions in this theory for values of $-V_M''/H_M^2$ close to 4 from both sides. The structure of the instanton solution is fully characterized by the Tanaka's curves $\mathcal{C}$. Since the potential (19) is an even function of Φ we need only one of the curves $\mathcal{C}^+$ and $\mathcal{C}^-$ ($\mathcal{C}^-$ is the mirror image of $\mathcal{C}^+$ with respect to vertical axis in the $(\bar{\Pi}, \bar{\Phi})$ plane). The instanton solutions are determined by the points at which $\mathcal{C}^+$ (or $\mathcal{C}^-$)

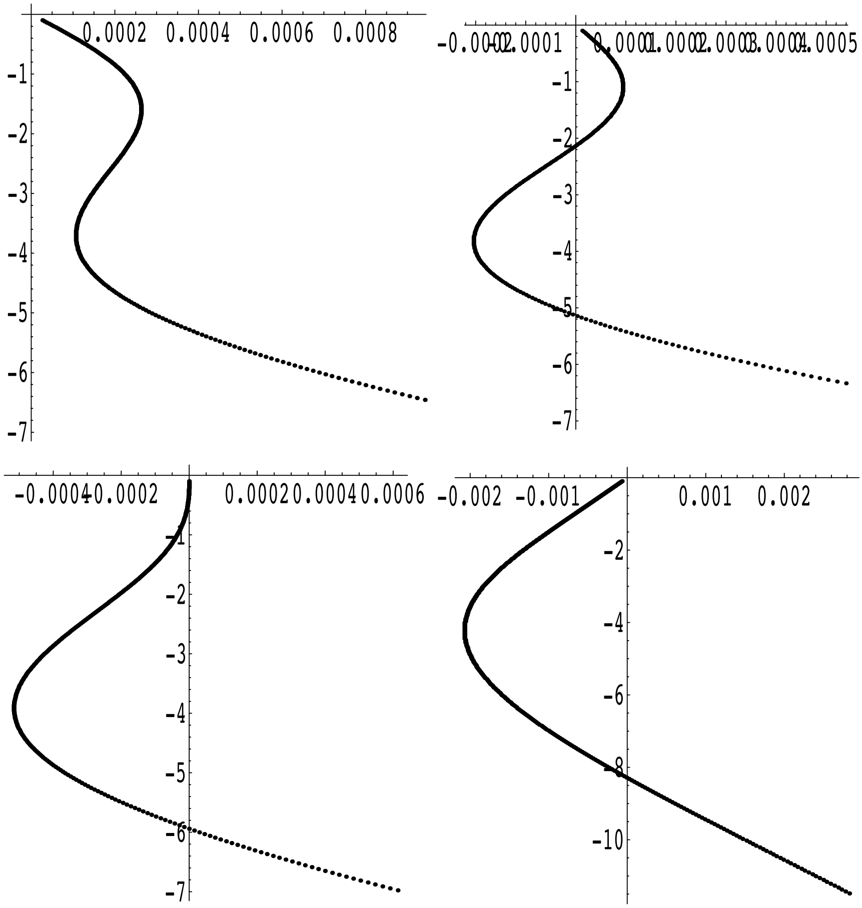

Fig. 2. The Tanaka's curves $\mathcal{C}^+$ (i.e. $\bar{\Pi}^+$ versus $\bar{\Phi}^+$) in the theory (19) with $g_4 = 300$. The graphs are plotted, from the left to the right and from top to bottom, for the values of $-V''_M/H_M^2 =$ 3.96, 3.98, 4.00 and 4.10 respectively.

crosses the vertical axis. The Tanaka's curves $\mathcal{C}^+$ with $-V''_M/H_M^2$ close to 4 and $g_4 = 300$ for the theory (19) are shown on the graphs in Fig. 2. These graphs tell us that for V''_M/H_M^2 close above 4 there is only one (no near-to-limit) CdL instanton, for $-V''_M/H_M^2 = 4$ lying between approximately 3.966 and 4 there are two CdL instantons, and for $-V''_M/H_M^2$ less than approximately 3.966 there are no CdL instantons. Finally, the structure of the instanton solutions in the theory (19) and the $-V''_M/H_M^2$-dependence of the instanton action are shown in Fig. 3.

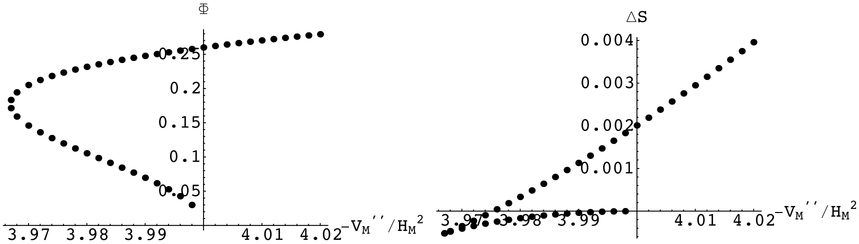

Fig. 3. Left graph: the initial value of the CdL instanton solution versus $-V_M''/H_M^2$ in the theory (19) with $g_4 = 300$. There are two bifurcation points in the parameter $-V_M''/H_M^2$ for solutions of the instanton equations. At $-V_M''/H_M^2 = 4$ the number of (nontrivial, i.e. no-HM-instanton) solutions changes from 1 to 2, and at a value approximately 3.966 the number of CdL instantons changes from 0 to 2. Right graph: the difference ΔS between the action of CdL and HM intanton (the action of the HM instanton is normalized to 2). Lower curve describes the subcritical near-to-limit CdL instanton and upper curve corresponds to the no-near-to-limit CdL instanton. These curves merge in the point $-V_M''/H_M^2 \approx 3.966$ mentioned above. For $-V_M''/H_M^2 \gtrapprox 3.975$ the no-near-to-limit instanton governs the vacuum decay, for the values below this the vacuum decay is governed by the HM instanton.

4. CONCLUSION

The false vacuum decay in a de Sitter universe has been investigated for near-to-critical values of the curvature of the effective potential. An approximate formula for the Euclidean action of the near-to-limit CdL instanton has been found by expanding the inflaton and the metric into to powers of the inflaton in a different way than in our previous work (Balek and Demetrian, 2005). We have focused on the case when the fourth derivative of the effective potential at its top has a subcritical value and $-V_M''/H_M^2$ is running from both sides around its critical value 4. We conclude that there is a range of the parameter $-V_M''/H_M^2$ less than 4 for which at least two CdL instantons exist. One of them is the near-to-limit instanton that can be described by the approximate formulas, together with its action, derived in the first part of the paper. The second instanton must exist because of the necessity to disconnect the energy curve from the potential when the starting point of the curve moves towards the true vacuum (Balek and Demetrian, 2004). The near-to-limit instanton in this case mediates the vaccum decay with a less probability than the related HM instanton. However, the vacuum decay is not governed by the HM instanton in this case but by the no-near-to-limit CdL instanton. On the other hand, we have shown on a concrete example that for sufficiently small values of $-V_M''/H_M^2$ the HM instanton has the least action from the three instantons in question and is to be considered as the instanton governing the vacuum decay.

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